SYST0002 - LECTURE #1 ESPACE D'ÉTAT EN 1D.

↳ DYNAMIQUE ⟺ ODE's.

$$\dot{x} \equiv \frac{dx}{dt} = f(x, \mu)$$
$$= a \cdot x + b \cdot \mu$$

→ PARAMÈTRES.

→ ENTRÉE(S)

→ VARIABLE(S)

$$\dot{x} = f(x)$$

SYSTÈME FERMÉ

$$\dot{x} = f(x, \mu)$$

SYSTÈME OUVERT.

$$\left(\begin{array}{l} \dot{x} = \sin(x) \\ \dot{x} = x^2 + \mu \\ \dot{x} = \sqrt{x} \end{array} \right.$$

$f(x)$ NON-LINÉAIRES!

$$\left(\begin{array}{l} \dot{x} = a \cdot x, \\ \dot{x} = a \cdot x + b\mu, \end{array} \right.$$

a PARAMÈTRE.

a, b PARAMÈTRES

LINÉAIRES

$\dot{x} = f(x)$ ~~RÉSOUDRE ? → x(t)~~

↳ MÉTHODES QUALITATIVES.

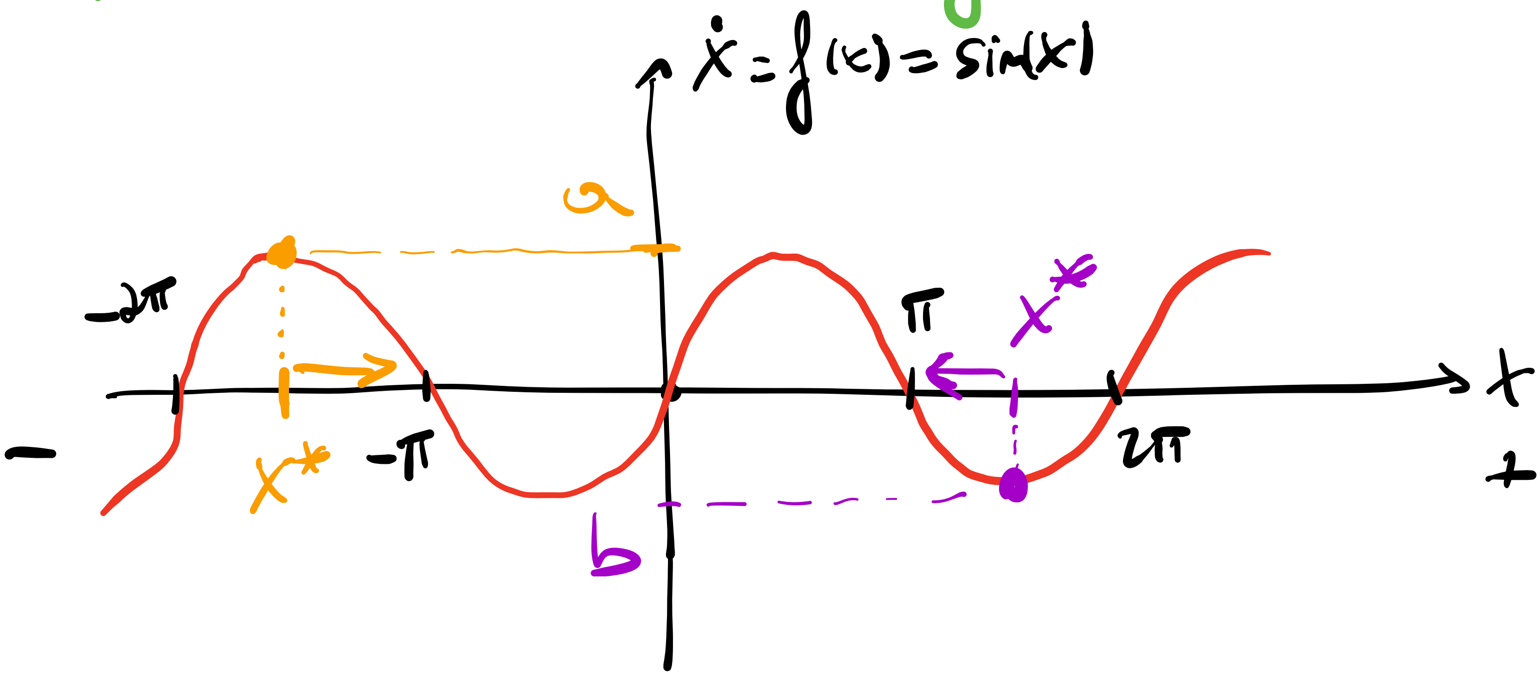
Ex : $\dot{x} = \sin(x)$ [NON LINÉAIRE].

↳ ① Si $x_0 = \frac{\pi}{4}$, VERS OÙ x VA CONVERGER ?

↳ ② POUR TOUTE C.I. ? $x \to$ $t \to +\infty$?

↳ PORTRAIT DE PHASE !

1D : TRACER $\dot{x}$ VS $x \Leftrightarrow f(x)$ VS x



↳ $f(x) = \sin(x)$ DÉTERMINE L'ÉVOLUTION TEMPORELLE DE x EN TOUT POINT.

- En x^*, $f(x^*) = a > 0$

  $\hookrightarrow \dot{x} > 0$, $\frac{dx}{dt} > 0$, $x \nearrow$

- En x^*, $f(x^*) = b < 0$

  $\hookrightarrow \dot{x} < 0$, $\frac{dx}{dt} < 0$, $x \searrow$

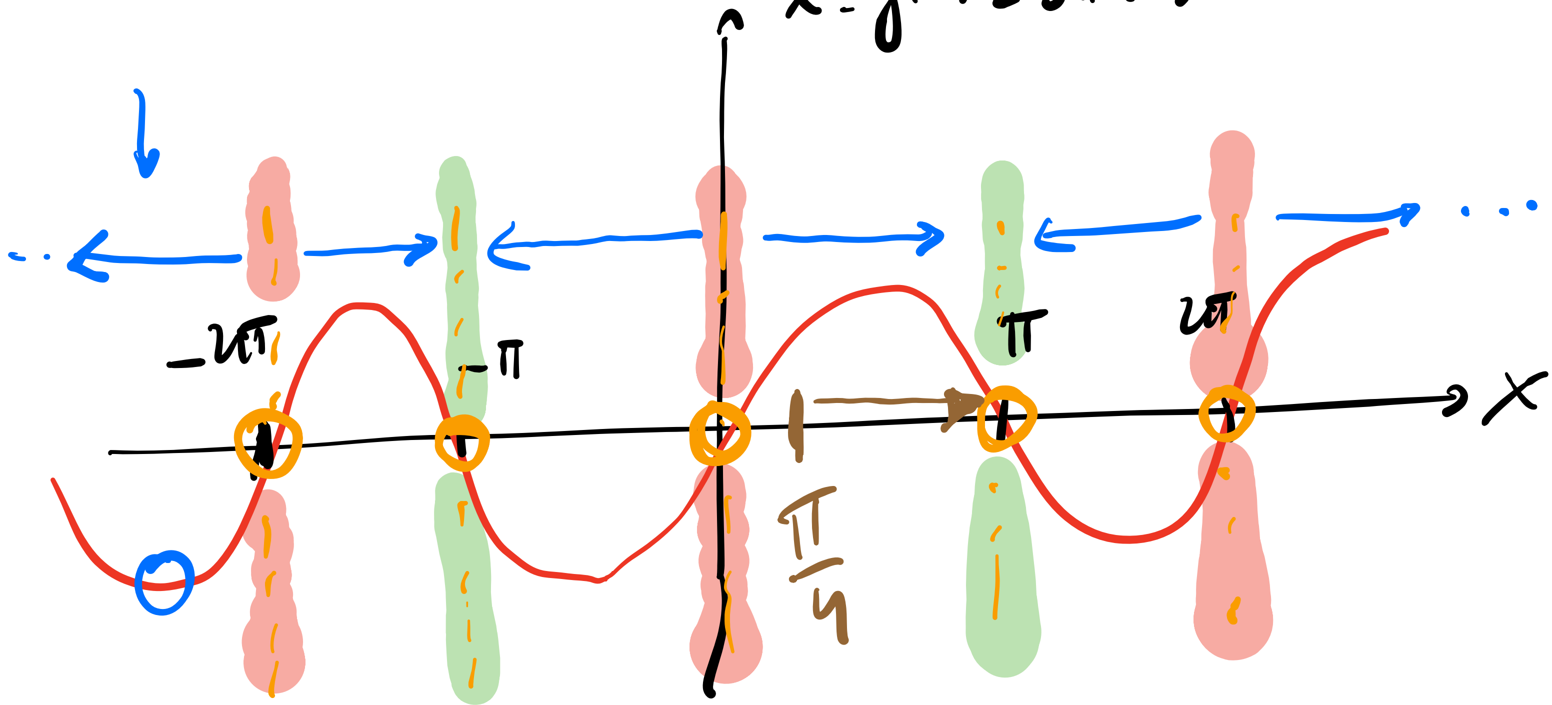
- En $\bar{x}$, $f(\bar{x}) = 0$

  $\hookrightarrow \dot{x} = 0$, $\frac{dx}{dt} = 0$, $x \longrightarrow$

EQUILIBRE / POINT FIXE.

ANALYSE DU PORTRAIT DE PHASE

$\dot{x} = f(x) = \sin(x)$



① POINTS FIXES ? $\dot{x} = f(\bar{x}) = 0.$

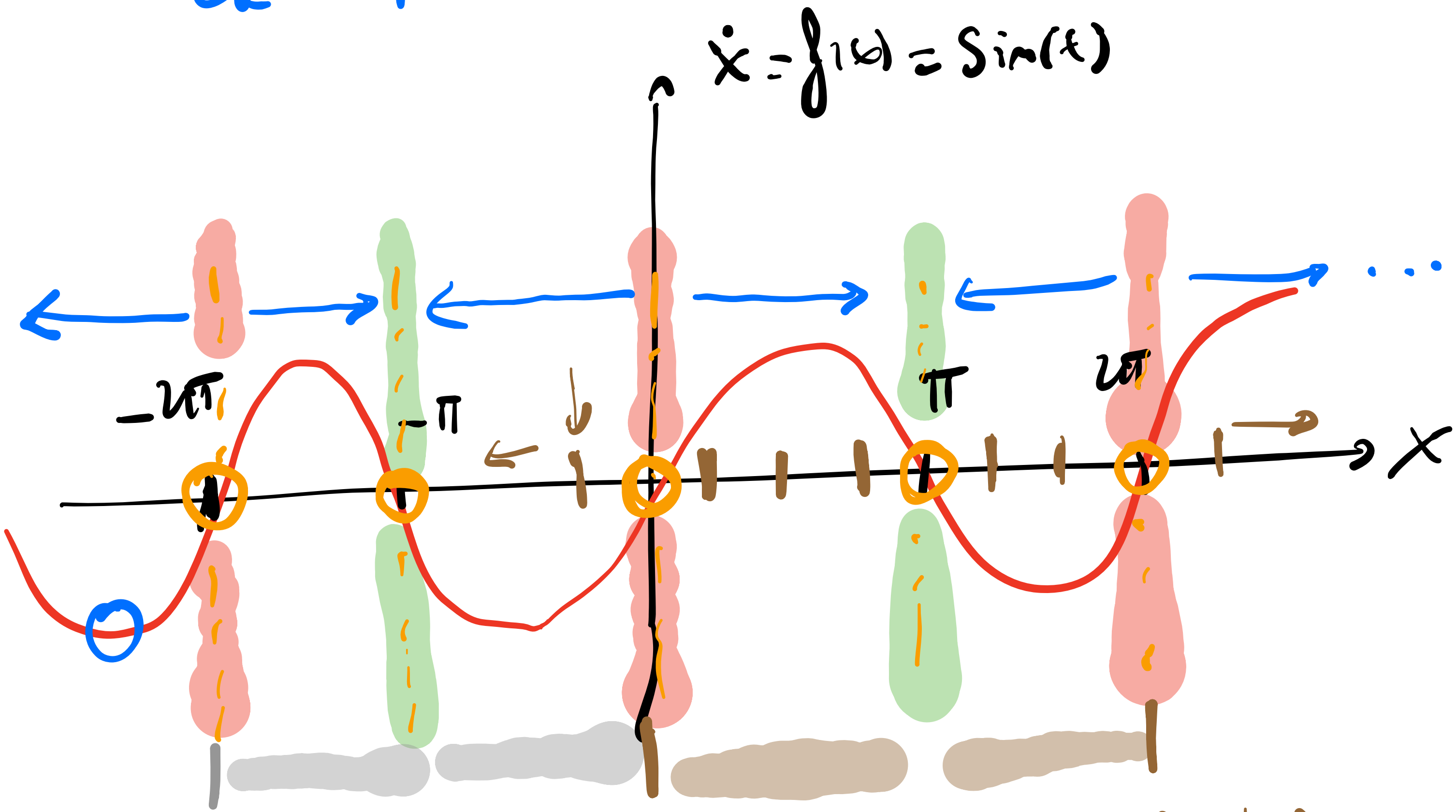
② ↳ POINTS REPULSIFS / INSTABLES.

↳ POINTS ATTRACTIFS / STABLES

↳ STABILITÉ DES POINTS FIXES.

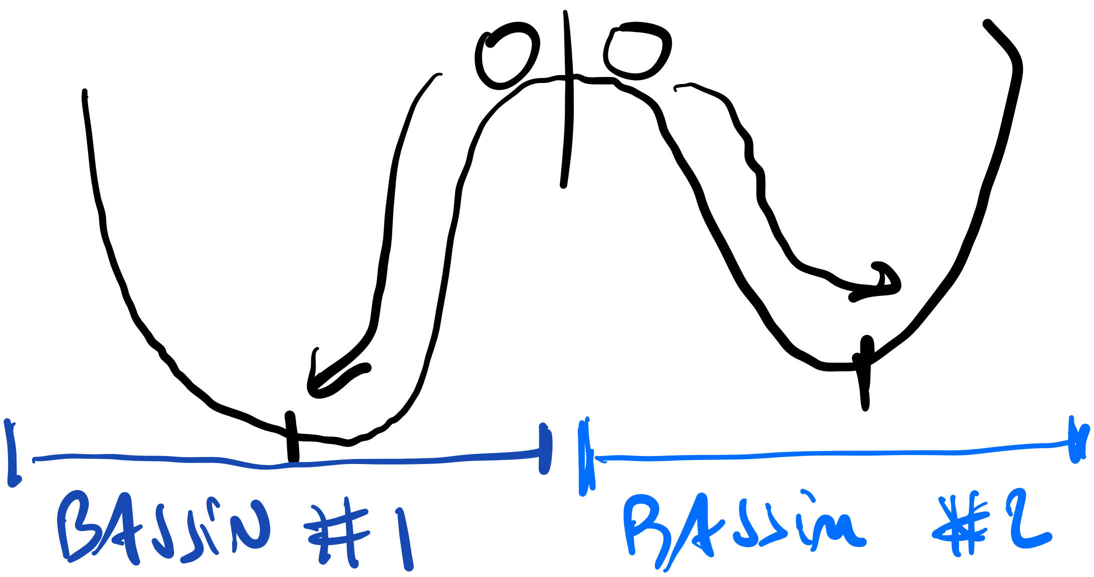
↳ POUR TOUTE C.I. ? BASSIN D'ATTRACTION [D'UN POINT FIXE STABLE]

= LES C.I. QUI VONT CONVERGER VERS CE POINT.

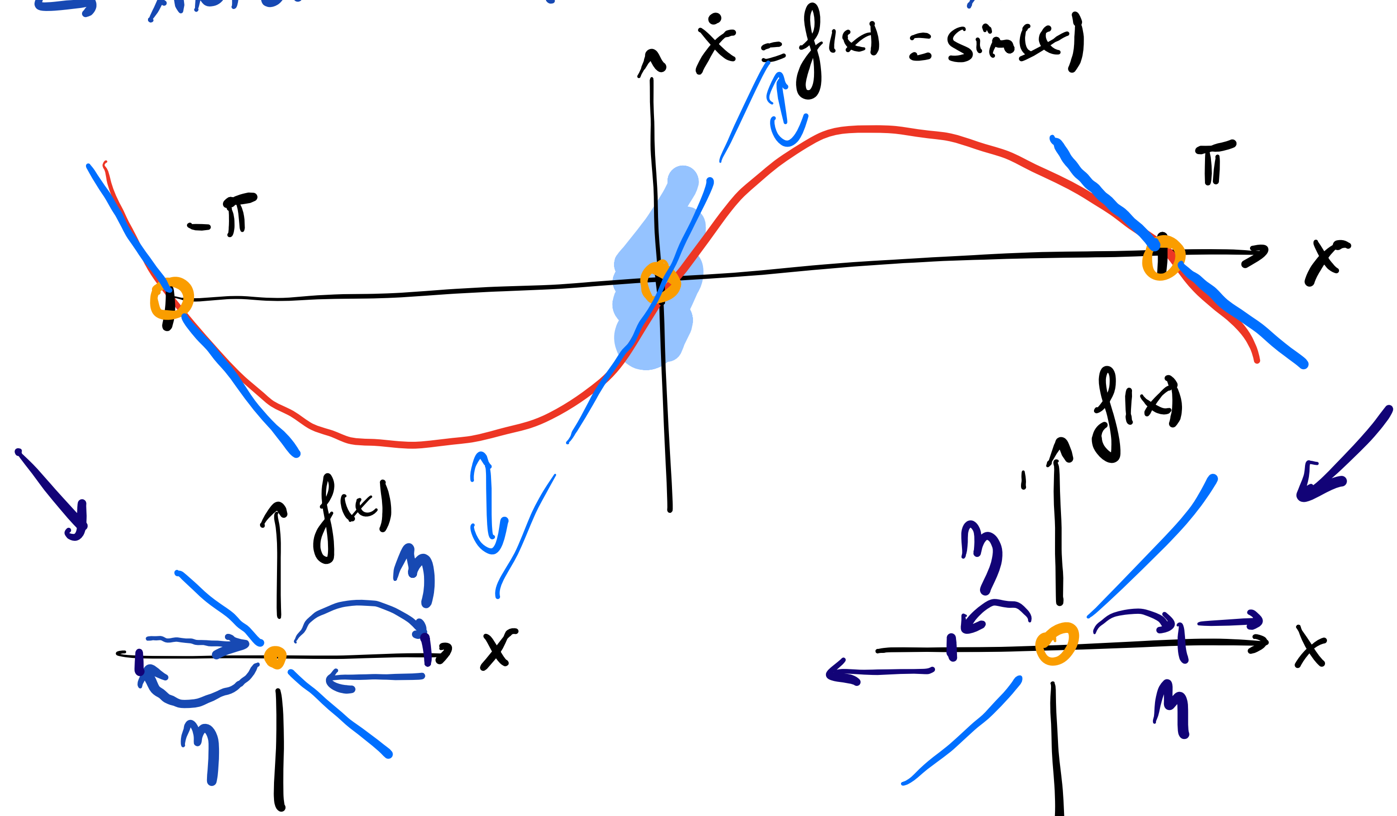


BASIN D'ATTRACTION DE $\bar{x} = -\pi$.

BASIN D'ATTRACTION DE $\bar{x} = \pi$



↳ ANALYSE LOCALE DE STABILITÉ.



⇒ PENTE < 0
→ STABLE !

⇒ PENTE > 0
→ INSTABLE !

PENTE $f(x)\big|_{\bar{x}}$

↳ LINÉARISATION !

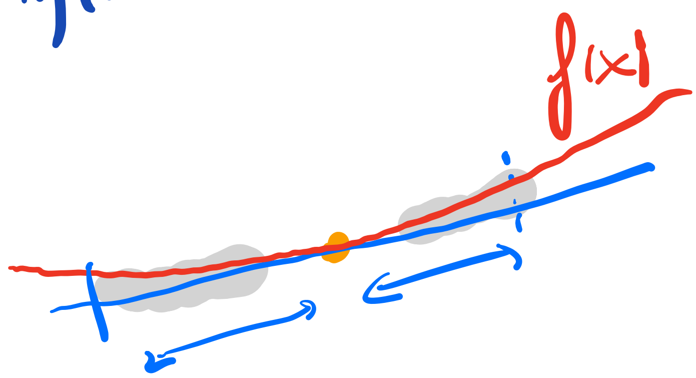
LOCAL !

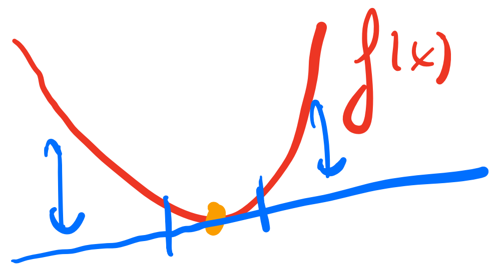
AUTOUR D'UN POINT !

LINÉARISATION AUTOUR DE $\bar{x}$:

↳ SI $\bar{x}$: POINT FIXE ET $\underline{\eta(t) = x(t) - \bar{x}}$

$\eta(t)$: PETITE PERTURBATION AUTOUR DE $\bar{x}$.





Ⓐ ↳ $\dot{\eta} = \frac{d}{dt}(x - \bar{x}) = \frac{dx}{dt} = f(x) = f(\bar{x}+\eta)$

⟹ EXPANSION DE TAYLOR

Ⓑ $f(\bar{x}+\eta) \simeq \underbrace{f(\bar{x})}_{=0} + \eta \cdot \frac{\partial f(\bar{x})}{\partial x} + O(\eta^2)$

TERME DOMINANT SI η PETIT

↳ $\dot{\eta} = f(\bar{x}+\eta) \simeq \eta \cdot \boxed{\frac{\partial f(\bar{x})}{\partial x}}$

PENTE DE $f(x)$ ÉVALUÉ EN $\bar{x}$. $= a$

$\simeq \eta . a$.

$\rightarrow \dot{\eta} = a.\eta \quad ; \quad a = \left.\frac{\partial f(x)}{\partial x}\right|_{\bar{x}}$

PARAMÈTRE.

$\hookrightarrow \dot{\eta} = f(\eta)$ où $f(\eta) = a.\eta$

$\rightarrow$ LINÉAIRE.

$$\begin{cases} \dot{\eta} = a.\eta \quad ; \quad a = \left.\frac{\partial f(x)}{\partial x}\right|_{\bar{x}} \\ \eta(0) = \eta_0 \end{cases}$$

$\hookrightarrow \eta(t) = \eta_0 e^{at}$

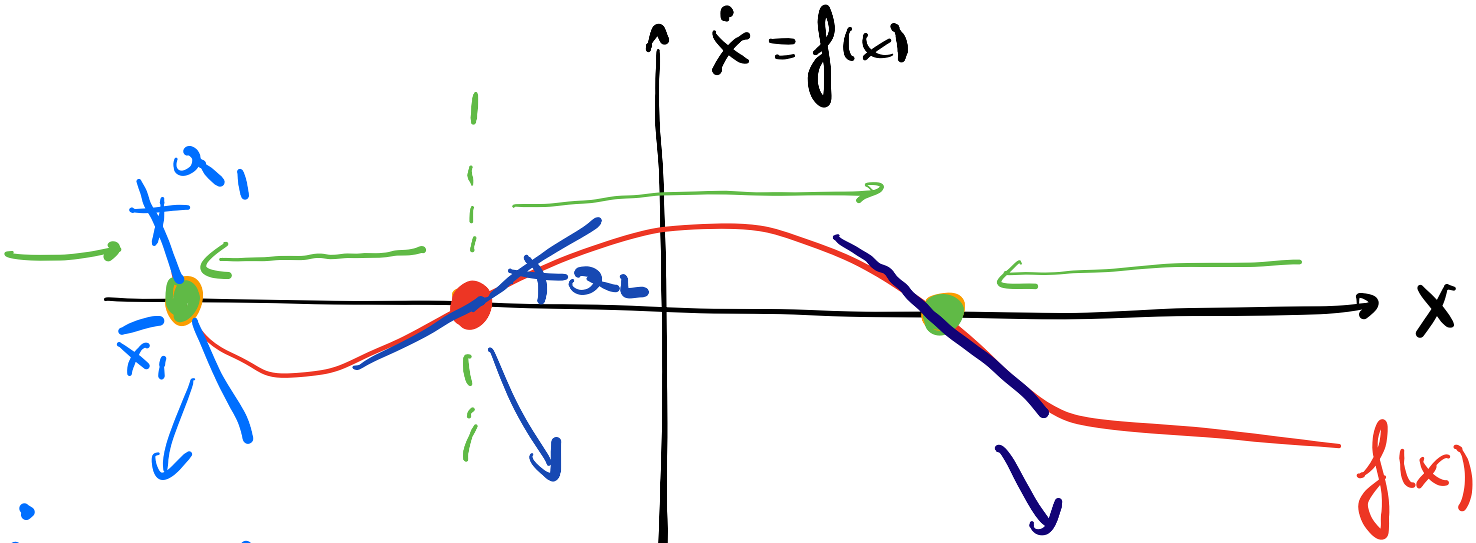
$a > 0$ $\rightarrow$ Exp $\nearrow$

$a < 0$ $\rightarrow$ Exp $\searrow$

! $a = 0$? $\rightarrow \left.\frac{\partial f(x)}{\partial x}\right|_{\bar{x}} = 0$

DOMINANT SI $a = 0$!

$$\hookrightarrow f(\bar{x}+\eta) \simeq \underbrace{f(\bar{x})}_{=0} + \underbrace{\eta . \left.\frac{\partial f(x)}{\partial x}\right|_{\bar{x}}}_{=0} + O(\eta^2)$$



$$\underbrace{\begin{gathered}\dot{\eta}_1 = a_1 \eta_1 \\ a_1 = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}_1}\end{gathered}} \neq \underbrace{\begin{gathered}\dot{\eta}_2 = a_2 \eta_2 \\ a_2 = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}_2}\end{gathered}} \neq \underbrace{\begin{gathered}\dot{\eta}_3 = a_3 \eta_3 \\ a_3 = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}_3}\end{gathered}}$$