SYST0002 – LECTURE #8 FONCTION DE TRANSFERT

$$U(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = H(s).U(s)$$

FONCTION DE TRANSFERT

EX: CONVERSION D'ENERGIE THERMIQUE VERS ELECTRIQUE DANS UNE TURBINE A VAPEUR [CONTRÔLE DE VALVES].

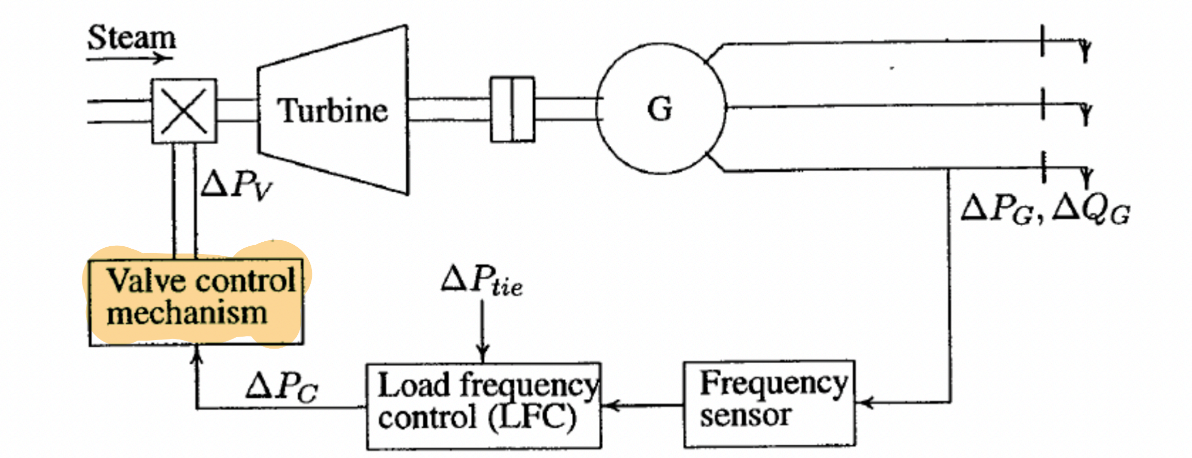


Fig 1. Schematic diagram of LFC of a synchronous generator

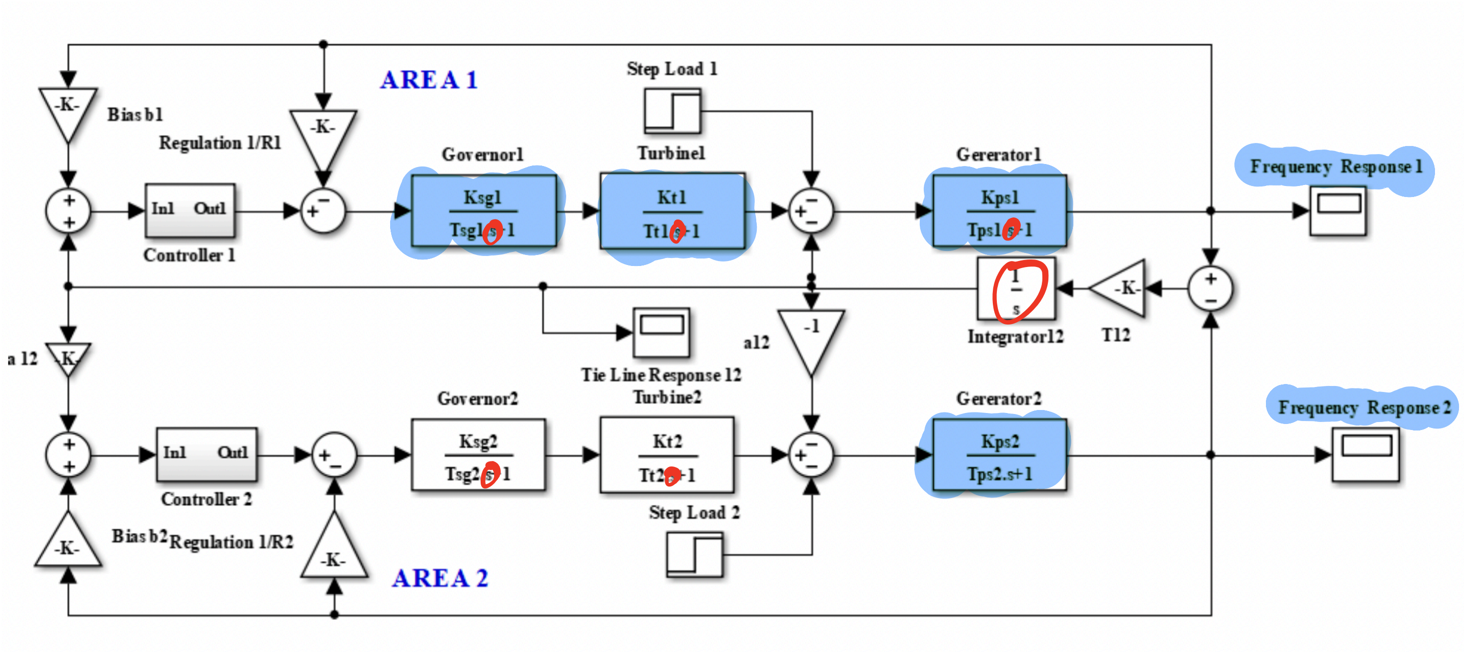


Fig 2. Transfer function model of two generating unit

TEMPOREL $u(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = u(t) * h(t)$

$\updownarrow \mathcal{L}$

FRÉQUENTIEL $U(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = U(s).H(s)$

↳ TRANSFORMÉE DE LAPLACE DE $h(t)$

$$\cdot\ h(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\underbrace{\sigma+j\omega}_{s}).e^{\overbrace{(\sigma+j\omega)}^{s}t}\, d\omega$$

$$.\ H(s) = \int_{-\infty}^{+\infty} h(t).e^{-st}\, dt \quad , \quad s = \sigma + j\omega$$

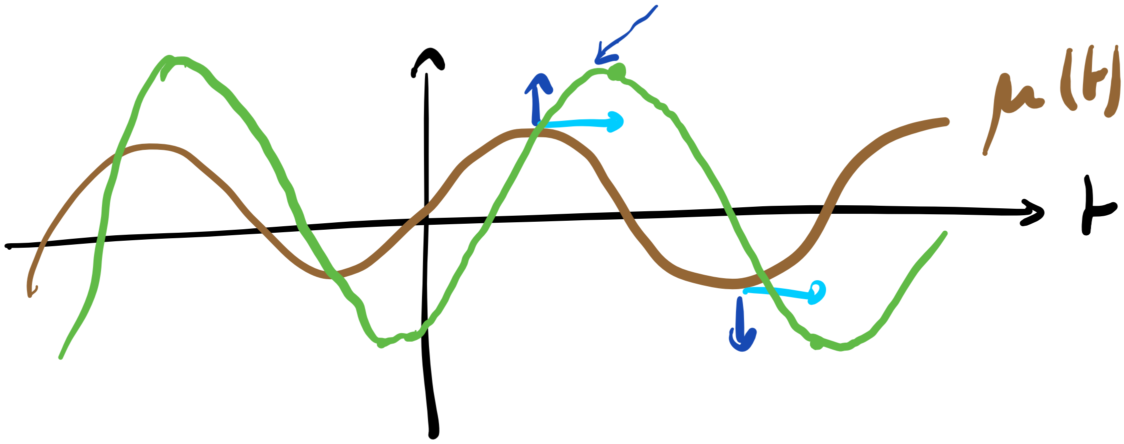
↳ $u(t) = e^{s_0 t} \longrightarrow \boxed{H(s)} \longrightarrow y(t) = e^{s_0 t}.H(s_0)$

$$\iff y(t) = e^{s_0 t}.\underbrace{|H(s_0)|}_{}.e^{j\underbrace{\angle H(s_0)}_{}}$$

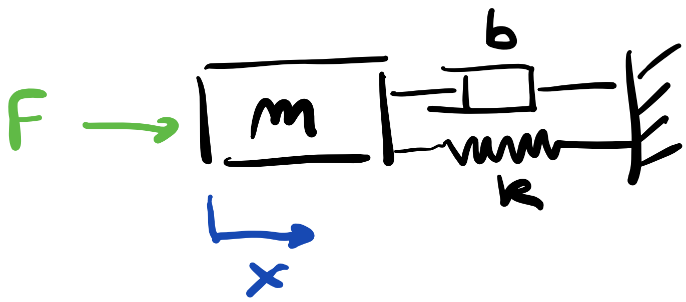
FRÉQUENCE N'EST PAS AFFECTÉE

AMPLITUDE
<1 ATTÉNUATION
>1 AMPLIFICATION

DÉPHASAGE
<0 : RETARD
>0 : AVANCE.



Ex:



$$\begin{cases} F = m\ddot{x} + b\dot{x} + kx \\ y = x \end{cases}$$

↳ CALCULER H(s) ? DOMAINE DE LAPLACE !

$$y(t) \xleftrightarrow{\mathcal{L}} Y(s)$$

$$\frac{dy(t)}{dt} \xleftrightarrow{\mathcal{L}} s.Y(s)$$

$$\frac{d^2y(t)}{dt^2} \xleftrightarrow{\mathcal{L}} s^2Y(s)$$

$$\vdots$$

$$\frac{d^ky(t)}{dt^k} \xleftrightarrow{\mathcal{L}} s^kY(s)$$

$$u(t) = m\ddot{x} + b\dot{x} + kx$$

$$\updownarrow \mathcal{L}$$

$$U(s) = ms^2X(s) + b.sX(s) + kX(s)$$

$$= \underbrace{\left[ms^2 + bs + k\right]}_{\text{POLYNÔME EN } s = \sigma + j\omega} X(s)$$

↳ H(s) ?

$$U(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = H(s).U(s)$$

↳ $\boxed{H(s) = \dfrac{Y(s)}{U(s)}}$ ←

$$y(t) = x(t) \xrightarrow{\mathcal{L}} Y(s) = X(s)$$

$$\hookrightarrow \quad U(s) = \left[ms^2 + bs + k\right] . Y(s)$$

$$\Rightarrow \boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{1}{ms^2 + bs + k}}$$

⚠ FONCTION RATIONNELLE !

$$\rightarrow \sum_{k=0}^{M} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k u(t)}{dt^k}, \quad a_M \neq 0$$

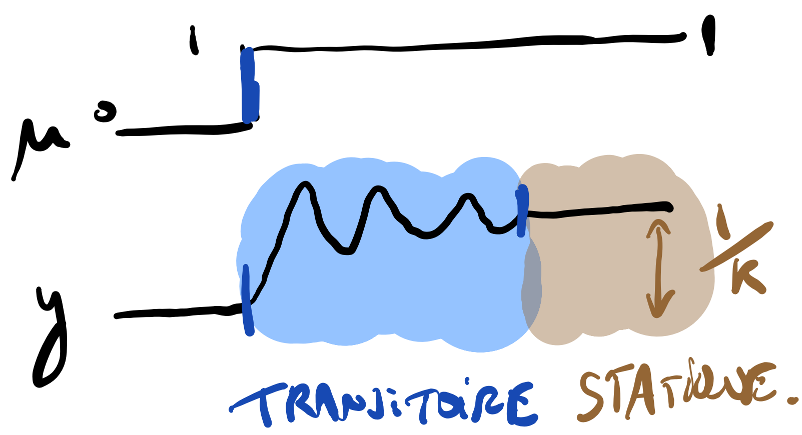
$$\updownarrow \mathcal{L}$$

$$\sum_{k=0}^{M} a_k . s^k Y(s) = \sum_{k=0}^{M} b_k . s^k U(s)$$

$$\hookrightarrow \quad H(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{k=0}^{M} b_k . s^k}{\sum_{k=0}^{M} a_k . s^k} = \frac{N(s)}{D(s)}$$

↳ ANALYSE DE H(s)

$$H(s) = \frac{1}{ms^2+bs+k}$$



① GAIN STATIQUE : H(s) AVEC $s=0$ [FREQ. 0]

↳ $$H(0) = \frac{1}{m\cdot 0^2 + b\cdot 0 + k} = \frac{1}{k}$$

② RÉPONSE DYNAMIQUE : STABLE ? OSCILLE ? VITESSE ?

↳ RAPPEL : ESPACE D'ÉTAT $x_1 = x$, $x_2 = \dot{x}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{b}{m} & -\frac{k}{m} \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix}}_{B} F$$

$$VP(A) \equiv \det(\lambda I - A) = 0$$

↳ $$\det\begin{pmatrix} \lambda + \frac{b}{m} & \frac{k}{m} \\ -1 & \lambda \end{pmatrix} = 0$$

$$\Leftrightarrow \lambda\left(\lambda + \frac{b}{m}\right) + \frac{k}{m} = 0$$

$$\Leftrightarrow \lambda^2 + \frac{b}{m}\lambda + \frac{k}{m} = 0$$

$\to \lambda_1$

$\to \lambda_2$

$$\rightarrow H(s) = \frac{1}{ms^2+bs+k} = \frac{1}{m}\cdot\frac{1}{s^2+\frac{b}{m}s+\frac{k}{m}}$$

↳ $s=p$ t.q. $D(p)=0 \equiv VP(A)$ PÔLES DE $H(s)$.

→ STABILITÉ : TOUS $\mathrm{Re}\{p\}<0$: STABLE.

TOUS $p\in\mathbb{R}$: MONOTONE.

SI $p_{1,2}=\sigma\pm j\omega$: OSCILLATOIRE.

$$\begin{cases}\dot{x} = Ax+Bu\\ y=Cx+Du\end{cases} \xleftrightarrow{\mathcal{L}} \begin{cases} sX=Ax+BU \\ Y=Cx+DU\end{cases}$$

$$H(s)=\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B+D$$

FONCTION DE TRANSFERT

$$H(s) = \frac{N(s)}{D(s)} = \frac{\sum_{k=0}^{M} b_k . s^k}{\sum_{k=0}^{M} a_k s^k}$$

- $s = p$ t.q. $D(p) = 0$: PÔLES DE $H(s)$
  - → STABILITÉ / DYNAMIQUE
  - → RÉPONSE LIBRE.
  - → RÉPONSE FRÉQUENTIELLE
- $s = z$ t.q. $N(z) = 0$: ZÉROS DE $H(s)$.
  - → RÉPONSE FORCÉE.
  - → RÉPONSE FRÉQUENTIELLE
- $H(0) = \frac{N(0)}{D(0)}$ GAIN STATIQUE

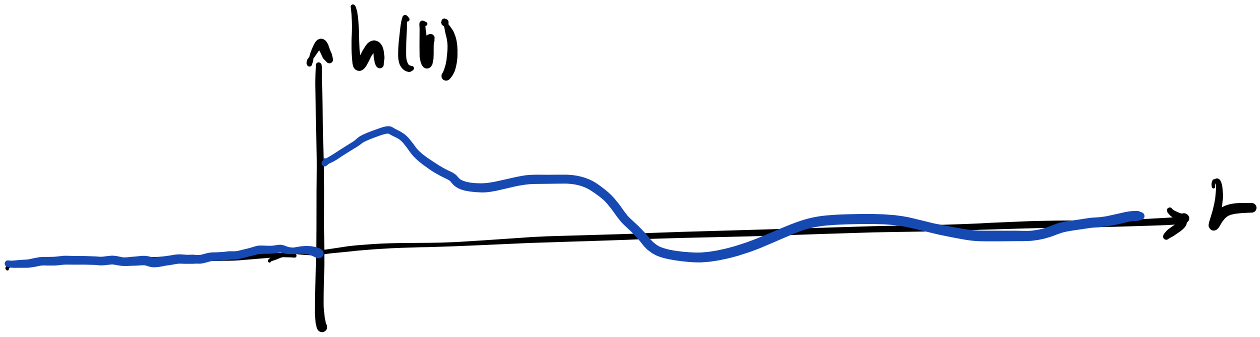
⚠ # PÔLES ≥ # ZÉROS

$$\frac{d^k y}{dt^k}, \frac{d^l u}{dt^l} \rightarrow k \geq l.$$

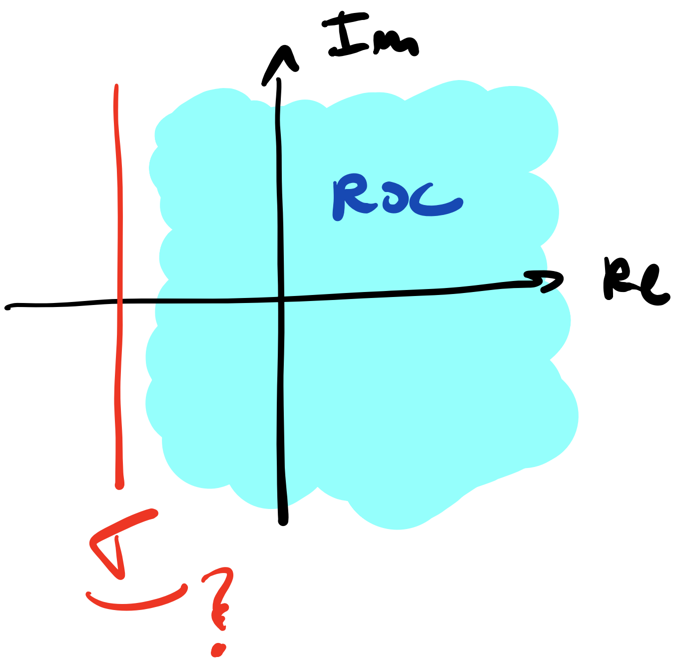
↳ $H(s) = \mathcal{L}\{h(t)\}$ → <u>ROC</u>!

→ ROC DE $H(s)$ D'UN SYSTÈME CAUSAL

$$h(t) = \underbrace{\mathbb{1}(t)}_{\text{CAUSAL}} \underbrace{\left[\sum_i C_i e^{\lambda_i t} \right]}_{\text{LTI.}}$$



SIGNAL A SUPPORT FINI A GAUCHE.



OBJECTIF : σ t.q. $h(t).e^{-\sigma t}$ CONVERGE

Ex : $\lambda_1 = -1$ → $p_1 = -1$

$\lambda_2 = -10$ → $p_2 = -10$ + CAUSAL

$$h(t) = \mathbb{1}(t).\left[C_1.e^{-t} + C_2.e^{-10t}\right]$$

↳ $$H(s) = \int_{-\infty}^{+\infty} h(t).e^{-st}\,dt$$

$$= \int_{-\infty}^{+\infty} \mathbb{1}(t)\left[\underbrace{C_1 e^{-t}}_{\text{I}} + \underbrace{C_2 e^{-10t}}_{\text{II}}\right] e^{-\sigma t}.e^{-j\omega t}\,dt$$

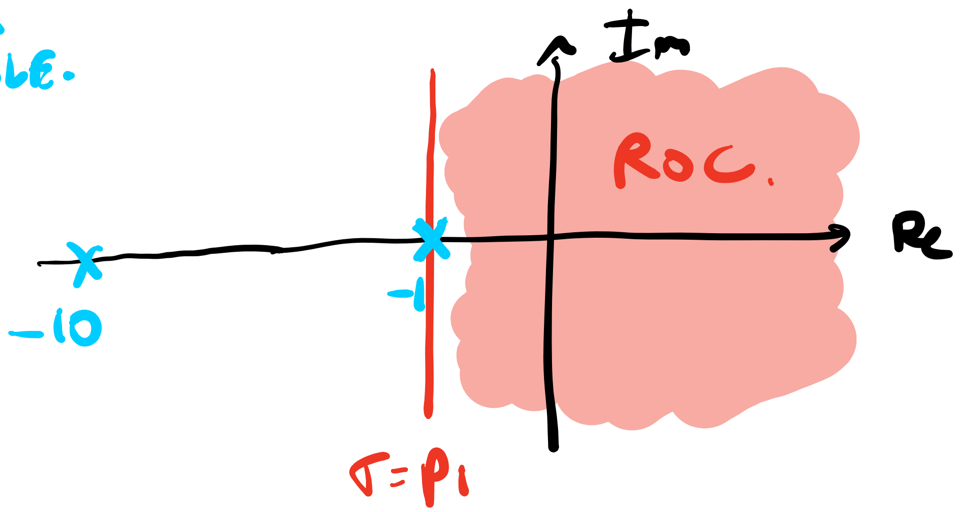
I $C_1 e^{-t}.e^{-\sigma t} = C_1.e^{-(1+\sigma)t}$: $\sigma > -1$

II $C_2 e^{-10t}.e^{-\sigma t} = C_2\, e^{-(10+\sigma).t}$: $\sigma > -10$

$\sigma > -1$

→ LIMITE DE ROC : $\sigma = -1 = \lambda_1 = p_1$.

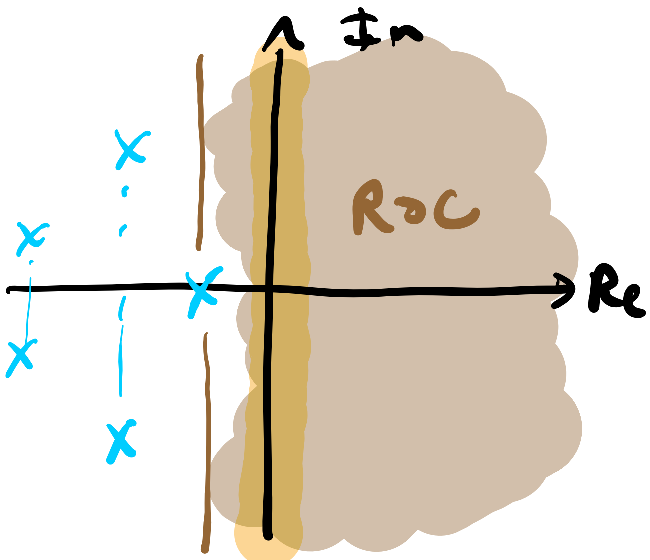
X : PÔLE.



LA ROC DE H(s) D'UN SYSTÈME LTI CAUSAL EST UN DEMI PLAN OUVERT À DROITE ET BORNÉ À GAUCHE PAR LE PÔLE DE H(s) AVEC LA PARTIE RÉELLE LA PLUS GRANDE.

[LE PÔLE LE PLUS À DROITE].

CAUSAL STABLE



x: POLE.

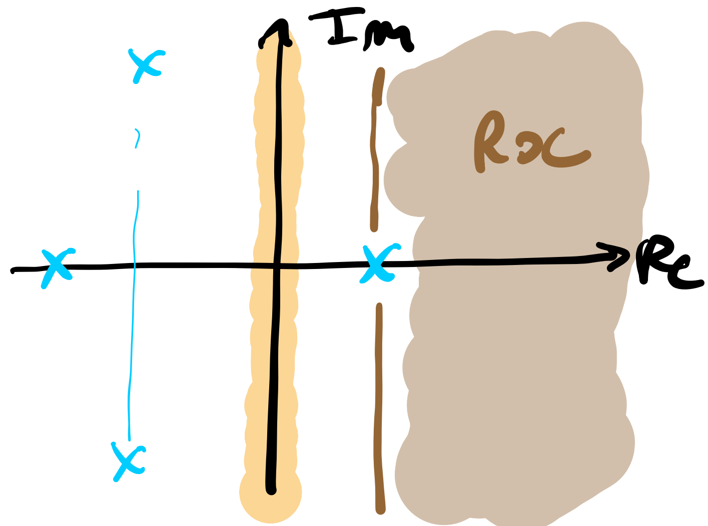
$s = \sigma + j\omega, \sigma = 0$

$s = j\omega$

$H(s) = H(j\omega)$

FOURIER.

CAUSAL INSTABLE.



$H(j\omega)$ NE CONVERGE PAS!