$$\dot{y} = \dot{u}$$

$$y = u + C$$

$y(0) \longrightarrow C = y(0) - u(0)$

$y(5) \longrightarrow C = y(5) - u(5)$

$\hookrightarrow y(5) = 1 \longrightarrow C = 1 - u(5)$

$$\Longrightarrow y = u + 1 - u(5)$$

(1) $sX(s) = AX(s) + BU(s)$

(2) $Y(s) = CX(s) + DU(s)$

$\Rightarrow H(s) = \dfrac{Y(s)}{U(s)}$?

(1) $sX(s) - AX(s) = BU(s)$

$\Leftrightarrow X(s)\,(s\mathbb{I} - A) = B\,U(s)$

$\Leftrightarrow X(s) = (s\mathbb{I} - A)^{-1} B\, U(s)$ (*)

(*) $\rightarrow$ (2) : $Y(s) = C(s\mathbb{I}-A)^{-1} B\, U(s) + DU(s)$

$$= \left[C(s\mathbb{I}-A)^{-1}B + D \right] U(s)$$

$$\Leftrightarrow H(s) = \frac{Y(s)}{U(s)} = C(s\mathbb{I}-A)^{-1}B + D$$

Exercice 1

Système : $\ddot{y} + \dot{y} - 2y = u$

a) $H(s)$? $\quad \dot{x}(t) \xleftrightarrow{\mathcal{L}} s\,X(s)$

$\Rightarrow s^2 Y(s) + s\,Y(s) - 2Y(s) = U(s)$

$\Leftrightarrow Y(s)\,(s^2 + s - 2) = U(s)$

$$\rightarrow \boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + s - 2}}$$

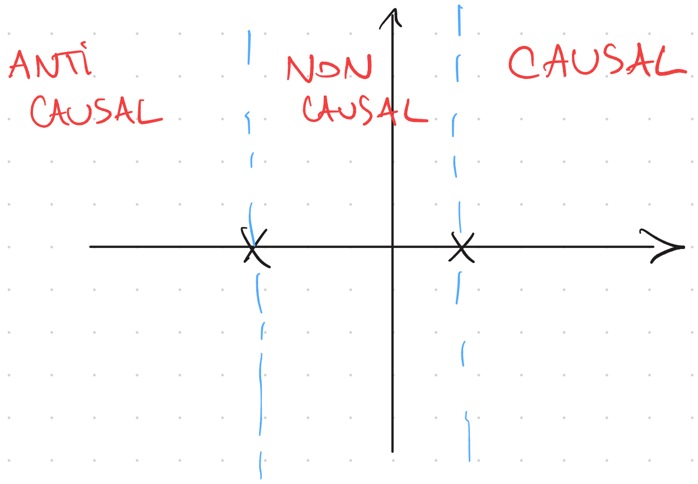
b) réponse impulsionnelle $h(t)$ et ROC

pour repasser en temporel → retomber sur des formes "$\frac{1}{s+a}$"

↳ DÉCOMPOSITION EN FRACTIONS SIMPLES

$$\frac{1}{s^2+s-2} = \frac{A}{s-1} + \frac{B}{s+2} \Leftrightarrow 1 = A(s+2) + B(s-1)$$

$$\begin{array}{ll} s=1 & \\ s=-2 & \end{array} \begin{cases} 1 = 3A & \Leftrightarrow A = 1/3 \\ 1 = -3B & \Leftrightarrow B = -1/3 \end{cases} \Rightarrow H(s) = \frac{1}{3(s-1)} - \frac{1}{3(s+2)}$$



$$H(s) = \frac{1}{3} \cdot \frac{1}{s-1} - \frac{1}{3} \frac{1}{s+2}$$

DE LA TABLE

$$e^{-at}\,\mathbb{I}(t) \xrightarrow{\mathcal{L}} \frac{1}{s+a} \quad \text{pour } \Re(s) > -a$$

$$-e^{-at}\,\mathbb{I}(-t) \xrightarrow{\mathcal{L}} \frac{1}{s+a} \quad \text{pour } \Re(s) < -a.$$

i) SYSTÈME CAUSAL : $ROC = \{s \in \mathbb{C} : \Re(s) > 1\}$

$$\Rightarrow h(t) = \frac{1}{3} e^{t}\,\mathbb{I}(t) - \frac{1}{3} e^{-2t}\,\mathbb{I}(t)$$

ii) SYSTÈME STABLE (NON-CAUSAL) : $ROC = \{s \in \mathbb{C} : -2 < \Re(s) < 1\}$

$$\Rightarrow h(t) = -\frac{1}{3} e^{t}\,\mathbb{I}(-t) - \frac{1}{3} e^{-2t}\,\mathbb{I}(t)$$

iii) SYSTÈME ANTICAUSAL : $ROC = \{s \in \mathbb{C} : \Re(s) < -2\}$

$$\Rightarrow h(t) = -\frac{1}{3} e^{t}\,\mathbb{I}(-t) + \frac{1}{3} e^{-2t}\,\mathbb{I}(-t)$$

EXERCICE 2

$$\ddot{y} = u - 3\dot{y} - 2y$$

i) A, B, C, D

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y \\ \dot{y} \end{pmatrix} \longrightarrow \begin{aligned} \dot{x}_1 &= 0x_1 + 1x_2 + 0u \\ \dot{x}_2 &= -2x_1 - 3x_2 + u \\ y &= 1x_1 + 0x_2 + 0u \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [1 \quad 0] \quad D = 0$$

ii) $H(s)$

$$\to \ddot{y} + 3\dot{y} + 2y = u \xrightarrow{\mathcal{L}} s^2 Y(s) + 3s\, Y(s) + 2\, Y(s) = U(s)$$

$$\longrightarrow \boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 3s + 2}}$$

iii) zéros, pôles et ROC

- zéros : pas de zéros
- pôles : $s^2 + 3s + 2 = (s+1)(s+2) \to$ pôles : $s = -1$ et $s = -2$
- ROC : système causal (énoncé) $\to$ ROC $= \{s \in \mathbb{C} : \mathcal{R}(s) > -1\}$

iv) stabilité ? OUI car la ROC comprend l'axe imaginaire

v) $h(t)$? $\to$ décomposition en fractions simples

$$H(s) = \frac{-1}{s+2} + \frac{1}{s+1} \Longrightarrow h(t) = \left(-e^{-2t} + e^{-t}\right) \mathbb{I}(t)$$

vi) $u(t) = e^{-3t}\,\mathbb{I}(t) \qquad y(0^-) = 1 \qquad \dot{y}(0^-) = 0$

$$Y(s) = Y_\ell(s) + Y_f(s) = C(s\mathbb{I}-A)^{-1}x[0] + H(s)\,U(s)$$

$$x[0] = \begin{pmatrix} y(0^-) \\ \dot{y}(0^-) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad U(s) = \frac{1}{s+3}$$

$$(s\mathbb{I}-A) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}$$

$$(s\mathbb{I}-A)^{-1} = \frac{1}{\det}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix} = \frac{1}{s^2+3s+2}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$Y(s) = \frac{1}{s^2+3s+2}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{s^2+3s+2}\cdot\frac{1}{s+3}$$

$$= \underbrace{\frac{s+3}{s^2+3s+2}}_{Y_\ell(s)} + \underbrace{\frac{1}{s^2+3s+2}\cdot\frac{1}{s+3}}_{Y_f(s)}$$

- $Y_\ell(s) = \dfrac{s+3}{s^2+3s+2} \longrightarrow Y_\ell(s) = \dfrac{2}{s+1} - \dfrac{1}{s+2}$

- $Y_f(s) = \dfrac{-1}{s+2} + \dfrac{1}{2(s+1)} + \dfrac{1}{2(s+3)}$

$$y(t) = y_\ell(t) + y_f(t) = \underbrace{2e^{-t}\mathbb{I}(t) - e^{-2t}\mathbb{I}(t)}_{y_\ell(t)} \underbrace{- e^{-2t}\mathbb{I}(t) + \frac{1}{2}e^{-t}\mathbb{I}(t) + \frac{1}{2}e^{-3t}\mathbb{I}(t)}_{y_f(t)}$$