## Exercice 1

$f(t) = \mathbb{I}(t) \qquad h(t) = 2e^{-t}\,\mathbb{I}(t) - 2e^{2t}\,\mathbb{I}(t)$

$y(t) = f(t) * h(t) = ?$

### Résolution analytique :

$\Rightarrow$ On repart de la formule : $y(t) = \int_{-\infty}^{+\infty} h(\tau)\, f(t-\tau)\, d\tau$

$$y(t) = \int_{-\infty}^{+\infty} \left(2e^{-\tau}\,\mathbb{I}(\tau) - 2e^{2\tau}\,\mathbb{I}(\tau)\right) \mathbb{I}(t-\tau)\, d\tau$$

(mise en évidence)

$$= \int_{-\infty}^{+\infty} \mathbb{I}(\tau)\left(2e^{-\tau} - 2e^{2\tau}\right)\mathbb{I}(t-\tau)\, d\tau$$

($\mathbb{I}(\tau) = 0$ pour $\tau < 0$)

$$= \int_{0}^{+\infty} \left(2e^{-\tau} - 2e^{2\tau}\right)\mathbb{I}(t-\tau)\, d\tau$$

$\hookrightarrow \mathbb{I}(t-\tau) = \begin{cases} 0 & \text{si } t < \tau \\ 1 & \text{si } t \geq \tau \end{cases}$

**2 cas :**

a) $t < 0$ : $y(t) = 0$

b) $t \geq 0$ : $y(t) = \int_0^t 2e^{-\tau} - 2e^{2\tau}\, d\tau$

$$= 2\left[-e^{-\tau} - \frac{1}{2}e^{2\tau}\right]_0^t$$

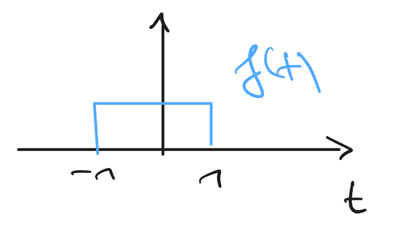
$$= 2\left(-e^{-t} - \frac{1}{2}e^{2t} + 1 + \frac{1}{2}\right)$$

$$= -2e^{-t} - e^{2t} + 3$$

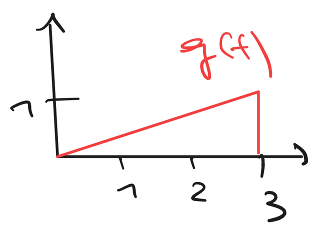
a) + b) : $\rightarrow$ $y(t) = (-2e^{-t} - e^{2t} + 3)\,\mathbb{I}(t)$

## Exercice 2

### Résolution graphique

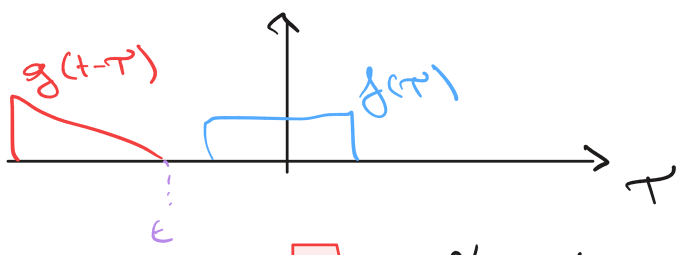



$f(t) = \begin{cases} 1 & \text{si } t \in [-1;1] \\ 0 & \text{sinon} \end{cases}$



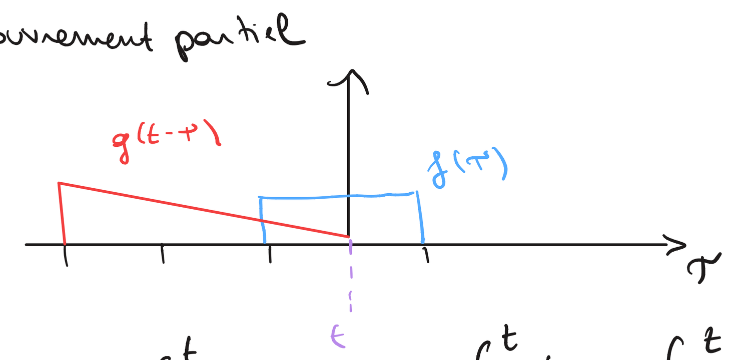

$g(t) = \begin{cases} t/3 & \text{si } t \in [0;3] \\ 0 & \text{sinon} \end{cases}$

- $t < -1$




$y(t) = \int_{-\infty}^{+\infty} f(\tau)\, g(t-\tau)\, d\tau = 0$ car l'aire de recouvrement est nulle

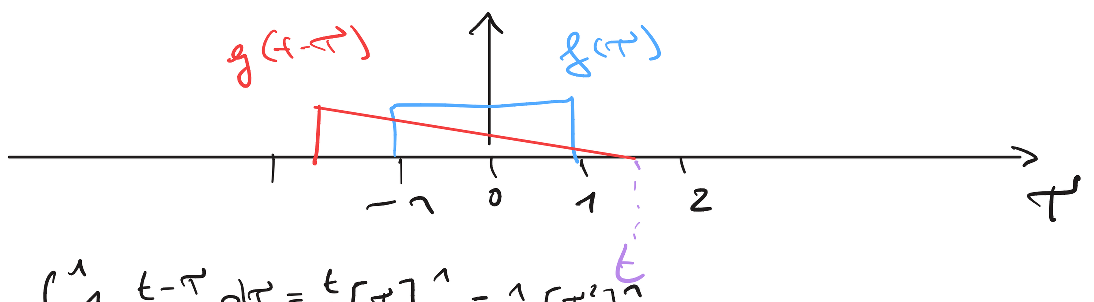
- $-1 < t < 1$ : Recouvrement partiel




$$y(t) = \int_{-\infty}^{+\infty} f(\tau)\, g(t-\tau)\, d\tau = \int_{-1}^{t} 1\, \frac{t-\tau}{3}\, d\tau = \int_{-1}^{t} \frac{t}{3}\, d\tau - \int_{-1}^{t} \frac{\tau}{3}\, d\tau$$

$$= \frac{t}{3}[\tau]_{-1}^{t} - \frac{1}{6}[\tau^2]_{-1}^{t} = \frac{t^2}{3} + \frac{t}{3} - \frac{t^2}{6} + \frac{1}{6} = \frac{1}{6}(t+1)^2$$

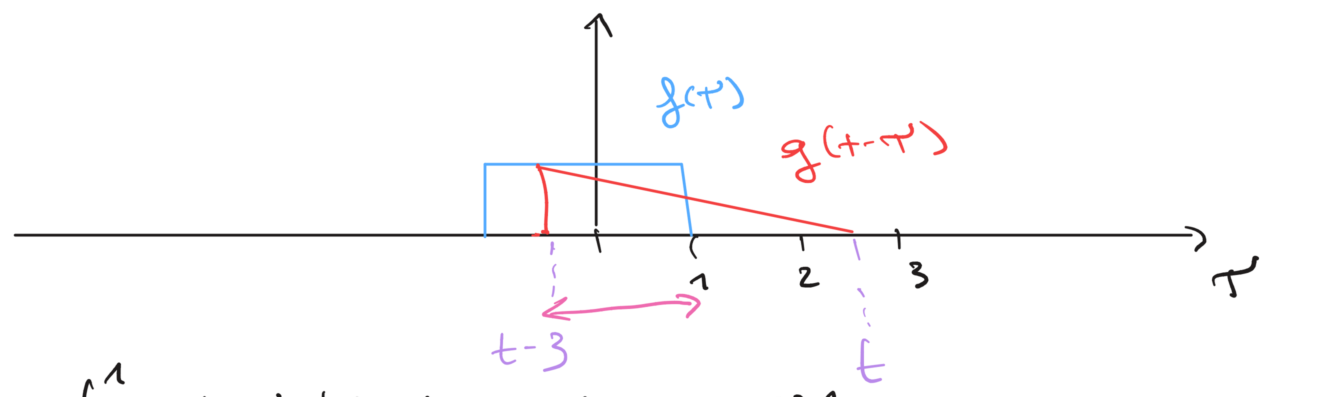
- $1 < t < 2$ : Recouvrement total




$$y(t) = \int_{-1}^{1} 1 \cdot \frac{t-\tau}{3}\, d\tau = \frac{t}{3}[\tau]_{-1}^{1} - \frac{1}{6}[\tau^2]_{-1}^{1}$$

$$= \frac{t}{3} + \frac{t}{3} - \frac{1}{6} + \frac{1}{6} = \frac{2t}{3}$$

- $2 < t < 4$ : Recouvrement partiel




$$y(t) = \int_{t-3}^{1} 1 \cdot \frac{t-\tau}{3}\, d\tau = \frac{t}{3}[\tau]_{t-3}^{1} - \frac{1}{6}[\tau^2]_{t-3}^{1}$$

$$= \frac{t}{3} - \frac{t(t-3)}{3} - \frac{1}{6} + \frac{1}{6}(t-3)^2 = \frac{-t^2 + 2t + 8}{6}$$

- $t > 4$ : Pas de recouvrement $\rightarrow$ $y(t) = 0$

### Réponse finale :

$$y(t) = \begin{cases} 0 & \text{si } t < -1 \text{ ou } t \geq 4 \\ \frac{1}{6}(t+1)^2 & \text{si } t \in [-1;1[ \\ \frac{2t}{3} & \text{si } t \in [1;2[ \\ \frac{-t^2+2t+8}{6} & \text{si } t \in [2;4[ \end{cases}$$

$\lim_{t \to -1^-} y(t) = 0 \quad = \quad \lim_{t \to -1^+} y(t) = 0$

$\lim_{t \to 1^-} y(t) = \frac{2}{3} \quad = \quad \lim_{t \to 1^+} y(t) = \frac{2}{3}$

$\lim_{t \to 2^-} y(t) = \frac{2.2}{3} = \frac{4}{3} \quad = \quad \lim_{t \to 2^+} y(t) = \frac{-4+4+8}{6} = \frac{8}{6} = \frac{4}{3}$

$\lim_{t \to 4^-} y(t) = \frac{-16+8+8}{6} = 0 \quad = \quad \lim_{t \to 4^+} y(t) = 0$